Differential form of Maxwell's equations

First equation

It states that the total electric flux ϕE passing through a closed hypothetical surface is equal to 1/ε0times

the net charge enclosed by the surface i.e, =q/ε0 or

=∫∫∫ρdV (1)

Apply Gauss's Divergence theorem to change L.H.S. of equation(1) from surface integral to volume

integral =∫v (∇.**D**)dV Substituting this

equation in equation (1), we get ∫∫∫ **(**∇.**D**)dV=∫∫∫ρdV

As two volume integrals are equal only if their integrands are equal.

Thus, ∇.**D**=ρ (2)

Equation (2) is the **Differential form of Maxwell's first equation.**

Second equation

It states that the total magnetic flux ϕm emerging through a closed surface is always equal to zero.

ϕm= =0 (3)

Apply Gauss's Divergence theorem =∫∫∫(∇.**B**)dV

Putting this in equation (3) ∫∫∫ (∇.**B**)dV =0

Thus , ∇.**B**=0 (4)

The equation (4) is **differential form of Maxwell's second equation.**

Third Equation

a) Itstates that,whenever magnetic flux linked with a circuit changes then induced electromotive force

(emf) is set up in the circuit. This induced emf lasts so long as the change in magnetic flux continues.

(b) The magnitude of induced emf is equal to the rate of change of magnetic flux linked with the circuit.

Therefore, induced emf= – dϕm/dt (5)

Where ϕm=∫∫**B.dS** (6)

Here negative sign is because of Lenz's law which states that the induced emf set up a current in such a

direction that the magnetic effect produced by it opposes the cause producing it.

Also, definition of emf states that emf is the closed line integral of the non-conservative electric field

generated by the battery.

That is emf= (7)

Putting equations (5) and (6), in equation (4) we get

= – ∫∫d**B**/dt.**dS** (8)

Apply Stoke's theorem to L.H.S. of equations (8) to change line integral to surface integral. That

is = ∫∫ (∇ **x E**).**dS**

By substituting above equation in equation(8), we get ∫∫ (∇ **x E**).**dS** =

-∫∫d**B**/dt.d**S**)

As two surface integral are equal only when their integrands are equal.

Thus ∇ **x E** = – d**B**/dt (9) This is

the differential form of Maxwell's 3rd equation.

Forth Equation(Displacement current, Modifying equation for the curl of

magnetic field to satisfy continuity equation)Modified Ampere's Circuital Law

Here the first question arises, **why there was need to modify Ampere′s circuital Law?**

To give answer to this question, let us first discuss Ampere's law (without modification)

Statement of Ampere′s circuital law (without modification): It states that the line integral of the

magnetic field **H** around any closed path or circuit is equal to the current enclosed by the path.

That is =I

Let the current is distributed through the surface with a current density